1. Solve each recurrence below and express $T(n)$ as the simplest $\Theta$ function of $n$.

   a. $T(n) = 25 T(n/5) + n^3$

   b. $T(n) = 64 T(n/4) + n^3$

   c. $T(n) = 81 T(n/3) + n^3$

   d. $T(n) = 2 T(n-1) + 1$

2. We are given an array $A$ that contains $n$ distinct values. We are told that the array $A$ had once been sorted, but then it was rotated by some unknown number of positions. Example: The array below was rotated either 7 positions left or 10 positions right, but assume these rotation distances are unknown to us. Design an efficient divide-and-conquer algorithm that returns the smallest value in $A$. Also write a recurrence for the worst-case running time, and solve the recurrence to obtain $T(n)$.

\[
\begin{array}{cccccccccccccccc}
19 & 23 & 29 & 31 & 37 & 41 & 43 & 47 & 53 & 59 & 2 & 3 & 5 & 7 & 11 & 13 & 17 \\
\end{array}
\]
3. First explain the details of how the $\Theta(n)$-time deterministic selection algorithm chooses the pivot value. Next suppose that we use these same steps to choose the pivot value in the quicksort algorithm. Write a recurrence for the worst-case running time of this version of quicksort, and solve the recurrence to obtain $T(n)$.

4. Among Bin sort, Bubble sort, Counting sort, Heap sort, Insertion sort, Merge sort, Quick sort, Radix sort, Selection sort: Which algorithm performs best in each case? (Justify your answers.)

   a. The input array is already sorted.

   b. The input array is in random order, and average case time is most important.

   c. Suppose that exchanges of the items in the array are very, very expensive. In other words, which method does the fewest “swaps” of the elements of the array?

   d. The worst case running time is the most important such as for a real time system.

   e. The input array consists of integers in the range $1...n^k$.

   f. The input array consists of bits (0s and 1s).
5. Give an $O(N)$ time algorithm to find the $\sqrt{N}$ values that are the closest (numerically) to the median value. Assume that you are given an unordered list $A$ of length $N$.

6. Some sorting algorithms, such as heap sort and in-place quick sort, are not usually stable. Explain a general method that will make any sorting algorithm stable.